Matric Exam Revisions For The Class Of 2020

MATHEMATICS P1

(2017, 2018, 2019)

QUESTION PAPERS & MEMOS

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MATHEMATICS P1

2017

QUESTION PAPER

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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2017

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 9x + 14 = 0$ (3)

1.1.2 $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2 Solve for x and y if:
$3x - y = 4$ and $x^2 + 2xy - y^2 = -2$ (6)

1.3 Given: $f(x) = x^2 + 8x + 16$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

[24]

QUESTION 2

2.1 Given the following quadratic number pattern: $5 \; ; -4 \; ; -19 \; ; -40 \; ; \ldots$

2.1.1 Determine the constant second difference of the sequence. (2)

2.1.2 Determine the n^{th} term (T_n) of the pattern. (4)

2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

2.2 The first three terms of an arithmetic sequence are $2k - 7$; $k + 8$ and $2k - 1$.

2.2.1 Calculate the value of the 15^{th} term of the sequence. (5)

2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)

[18]

QUESTION 3

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.

3.1 If $T_1 + T_2 = 2$, write down an expression for a in terms of r. (2)

3.2 Calculate the values of a and r. (6)

[8]

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1\ ;\ \frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (3)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)

4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2)

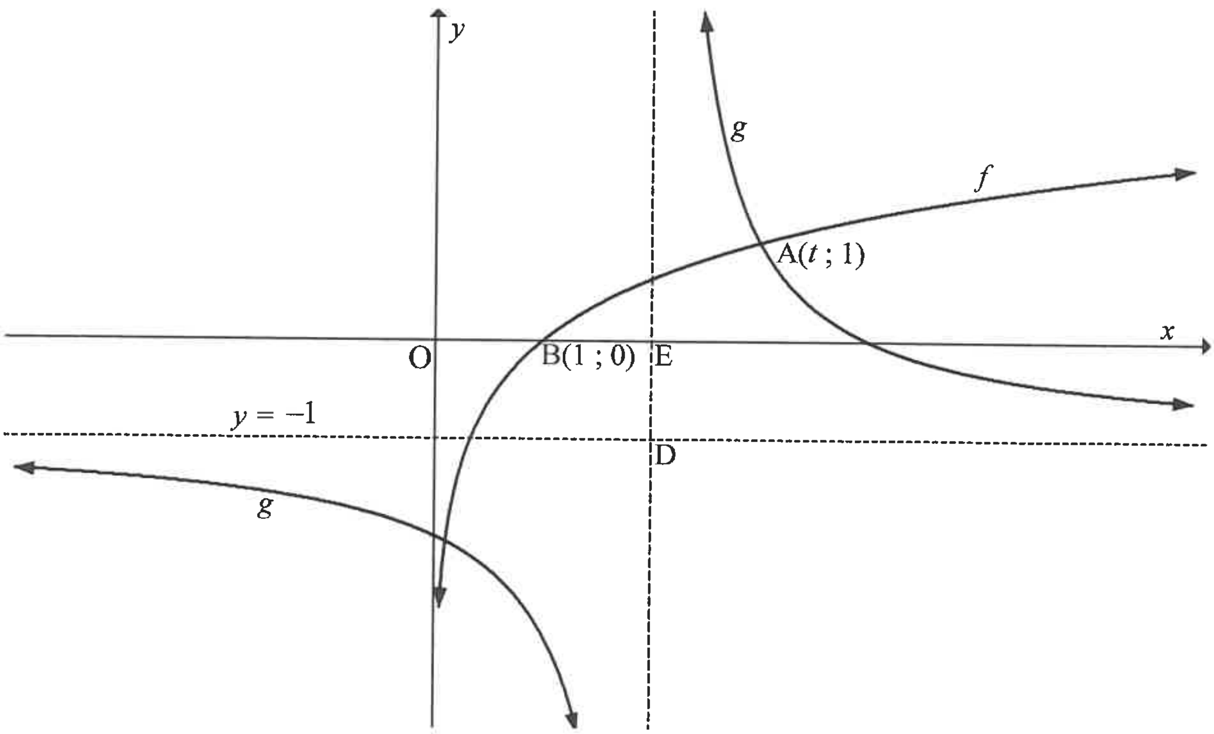
4.7 Write down the values of x for which $f(x).g(x) \leq 0$. (3)

[23]

QUESTION 5

The diagram below shows the graphs of $g(x)=\frac{2}{x+p}+q$ and $f(x)=\log_3 x$.

- $y=-1$ is the horizontal asymptote of g.
- B(1 ; 0) is the x-intercept of f.
- A(t ; 1) is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB = BE.



5.1	Write down the range of g.	(2)
5.2	Determine the equation of g.	(2)
5.3	Calculate the value of t.	(3)
5.4	Write down the equation of f^{-1}, the inverse of f, in the form $y=\ldots$	(2)
5.5	For which values of x will $f^{-1}(x)<3$?	(2)
5.6	Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient.	(3)

[14]

QUESTION 6

6.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5)

6.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

6.2.1 Calculate Piet's monthly instalment. (4)

6.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)

[15]

QUESTION 7

7.1 Given: $f(x) = 2x^2 - x$

Determine $f'(x)$ from first principles. (6)

7.2 Determine:

7.2.1 $D_x[(x+1)(3x-7)]$ (2)

7.2.2 $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}\pi$ (4)

[12]

QUESTION 8

Given: $f(x)=x(x-3)^2$ with $f'(1)=f'(3)=0$ and $f(1)=4$

8.1 Show that f has a point of inflection at $x=2$. (5)

8.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

8.3 For which values of x will $y=-f(x)$ be concave down? (2)

8.4 Use your graph to answer the following questions:

8.4.1 Determine the coordinates of the local maximum of h if $h(x)=f(x-2)+3$. (2)

8.4.2 Claire claims that $f'(2)=1$.

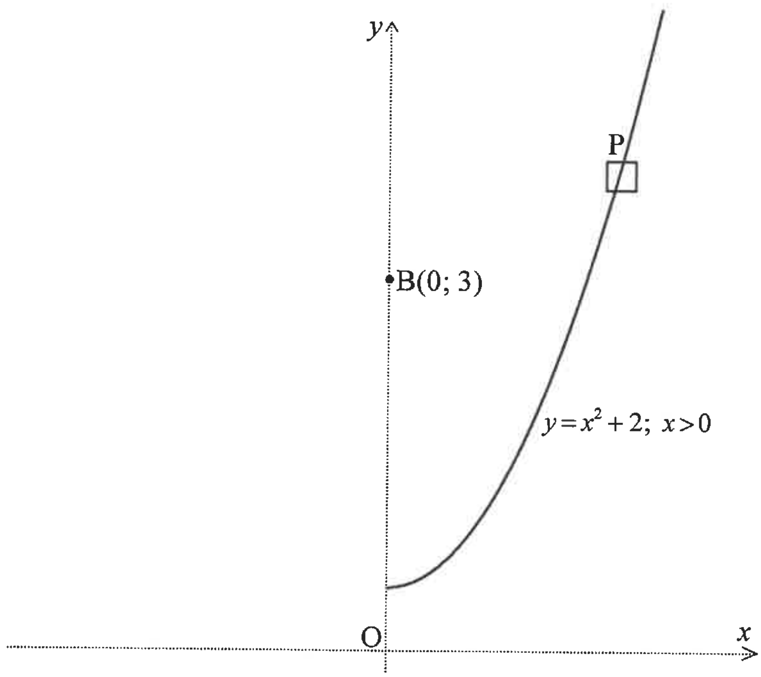
Do you agree with Claire? Justify your answer. (2)

[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y=x^2+2$, $x\geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

QUESTION 10

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

10.1 Draw a Venn diagram to illustrate the information above. (4)

10.2 Calculate the value of x. (2)

10.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)
[8]

QUESTION 11

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR 45789.

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

11.1 How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)

11.2 Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3)
[6]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r};\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

InΔABC: $\quad \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta \mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

LIFE SCIENCES P2

2017

MEMO

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NATIONAL SENIOR CERTIFICATE
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GRADE 12/*GRAAD* 12

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE:* *150*

This memorandum consists of 22 pages.
Hierdie memorandum bestaan uit 22 bladsye.

NOTE:

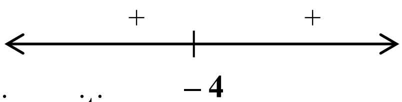
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in ALL aspects of the marking guidelines.

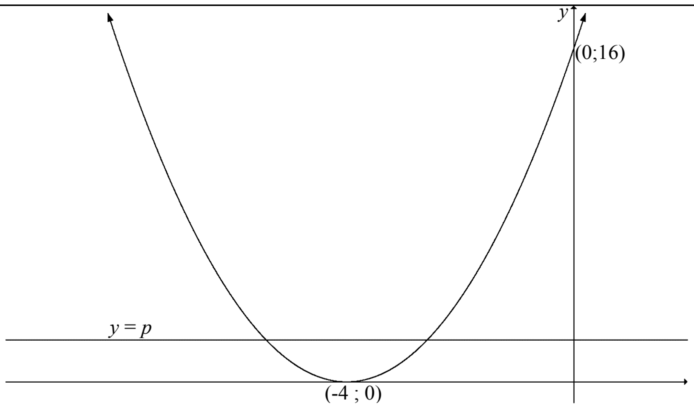
LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION/*VRAAG* 1

1.1.1	$x^2+9x+14=0$ $(x+7)(x+2)=0$ $x=-7$ or $x=-2$	✓ factors ✓ $x=-7$ ✓ $x=-2$ (3)
1.1.2	$4x^2+9x-3=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-9\pm\sqrt{9^2-4(4)(-3)}}{2(4)}$ $=\frac{-9\pm\sqrt{129}}{8}$ $x=0{,}29$ or $x=-2{,}54$ **OR/*OF*** $x^2+\frac{9}{4}x+\frac{81}{64}=\frac{3}{4}+\frac{81}{64}$ $\left(x+\frac{9}{8}\right)^2=\frac{129}{64}$ $x+\frac{9}{8}=\pm\frac{\sqrt{129}}{8}$ $x=\frac{-9\pm\sqrt{129}}{8}$ $x=0{,}29$ or $x=-2{,}54$	✓ substitution ✓ simplification ✓ $x=0{,}29$ ✓ $x=-2{,}54$ **OR/*OF*** ✓for adding $\frac{81}{64}$ on both sides ✓ simplification ✓ $x=0{,}29$ ✓ $x=-2{,}54$ (4)
1.1.3	$\sqrt{x^2-5}=2\sqrt{x}$ $x^2-5=4x$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=5$ or $x=-1$ $x=5$	✓ $x^2-5=4x$ ✓ standard form ✓ both answers ✓ select $x=5$ (4)

1.2	$3x - y = 4$ $y = 3x - 4$ $x^2 + 2xy - y^2 = -2$ $x^2 + 2x(3x-4) - (3x-4)^2 = -2$ $x^2 + 6x^2 - 8x - (9x^2 - 24x + 16) = -2$ $7x^2 - 8x - 9x^2 + 24x - 16 = -2$ $-2x^2 + 16x - 14 = 0$ $x^2 - 8x + 7 = 0$ $(x-7)(x-1) = 0$ $x = 1$ or $x = 7$ $y = 3(1) - 4$ $y = 3(7) - 4$ $y = -1$ or $y = 17$ **OR/*OF*** $3x - y = 4$ $x = \frac{y+4}{3}$ $x^2 + 2xy - y^2 = -2$ $x^2 + 2xy - y^2 = -2$ $\left(\frac{y+4}{3}\right)^2 + 2\left(\frac{y+4}{3}\right)y - y^2 = -2$ $y^2 + 8y + 16 + 6y^2 + 24y - 9y^2 = -18$ $-2y^2 + 32y + 34 = 0$ $y^2 - 16y - 17 = 0$ $(y-17)(y+1) = 0$ $y = -1$ or $y = 17$ $x = \frac{-1+4}{3}$ $x = \frac{17+4}{3}$ $x = 1$ or $x = 7$	✓ y subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓x-values ✓y-values **OR/*OF*** ✓x subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓y-values ✓x-values (6)
1.3.1	$x^2 + 8x + 16 > 0$ $(x+4)(x+4) > 0$ $x \in R,\ x \neq -4$ **or** $x \in (-\infty; -4)$ or $x \in (-4; \infty)$ **or** $x < -4$ or $x > -4$ **OR/*OF*** $x^2 + 8x + 16 > 0$ $(x+4)(x+4) > 0$  The function values remain positive $x \in R,\ x \neq -4$	✓ $(x+4)(x+4)$ ✓✓ any one of the solutions **OR/*OF*** ✓ $(x+4)(x+4)$ ✓✓ any one of the solutions (3)

<table>
<tr><td>1.3.2</td><td>

For two negative unequal roots:
$0 < p < 16$

OR/OF
$x^2+8x+16=p$
$x^2+8x+16-p=0$
$0<16-p<16$
$-16<-p<0$
$0<p<16$
OR/OF
$x^2+8x+16-p=0$
$x=\frac{-8\pm\sqrt{64-4(16-p)}}{2}$
$0<64-4(16-p)<64$
$0<4p<64$
$0<p<16$
OR/OF
$x^2+8x+16=p$
$x^2+8x+16-p=0$
Roots are real and unequal:
$8^2-4(16-p)>0$
$4p>0$
$p>0$
Roots are: $\frac{-8\pm\sqrt{4p}}{2}$
For both roots to be negative:
$\sqrt{4\mathrm{p}}<8$
$4p<64$
$p<16$
$0<p<16$</td><td>✓ 0
✓ 16
✓✓ $0<p<16$ (4)

OR/OF

✓ 0
✓ 16
✓✓ $0<p<16$ (4)

✓ 0
✓ 16
✓✓ $0<p<16$ (4)

✓ 0
✓ 16
✓✓ $0<p<16$ (4)
[24]</td></tr>
</table>

QUESTION/*VRAAG* 2

2.1.1	5 –4 –19 –40
	   –9 –15 –21
	     –6 –6
	
	first differences: –9; –15; –21
	second difference = – 6
		✓ first differences
		✓ – 6
		(2)

2.1.2	$T_n = an^2 + bn + c$ $a = \frac{\text{second difference}}{2} = -3$ $3a + b = -9$ $3(-3) + b = -9$ $b = 0$ $a + b + c = 5$ $-3 + 0 + c = 5$ $c = 8$ $T_n = -3n^2 + 8$ **OR/*OF*** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$ $= 5 + (n-1)(-9) + \frac{(n-1)(n-2)(-6)}{2}$ $= 5 - 9n + 9 - 3n^2 + 9n - 6$ $T_n = -3n^2 + 8$	✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ **OR/*OF*** ✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ (4)
2.1.3	$-3n^2 + 8 = -25\,939$ $-3n^2 = -25947$ $n^2 = 8649$ $n = -93 \quad or \quad n = 93$ The 93rd term has a value of –25 939	✓ $T_n = -25\,939$ ✓ $n^2 = 8649$ ✓ answer (3)

2.2.1	$2k-7\ ;\ k+8$ and $2k-1$ $k+8-(2k-7)=2k-1-(k+8)$ $-k+15=k-9$ $2k=24$ $k=12$ $2k-7; k+8$ and $2k-1$ $17\ ;20\ ;23.......$ $d=3$ $T_{15}=17+14(3)$ $=59$	✓ $k+8-(2k-7)=2k-1-(k+8)$ ✓$k=12$ ✓17 ✓ $d=3$ ✓$T_{15}=59$ (5)
2.2.2	Sequence is 17 ; 20 ; 23 ; 26 ; 29 ; 32 ……. Every alternate term of the sequence will be even / *Elke tweede term van die ry sal ewe wees* $20+26+32+.......$ $S_{30}=\frac{30}{2}[2(20)+(29)(6)]$ $=15[40+174]$ $=3210$ **OR/*OF*** $T_{30}=20+29(6)$ $=94$ $S_{30}=\frac{30}{2}(20+194)$ $=3210$	✓ $20+26+32+.......$ ✓$a=20\ \ d=6$ ✓subst into correct formula ✓ answer (4) ✓ $a=20\ \ d=6$ ✓ $T_{30}=94$ ✓ $S_{30}=\frac{30}{2}(20+194)$ ✓ answer (4) **[18]**

QUESTION/*VRAAG* 3

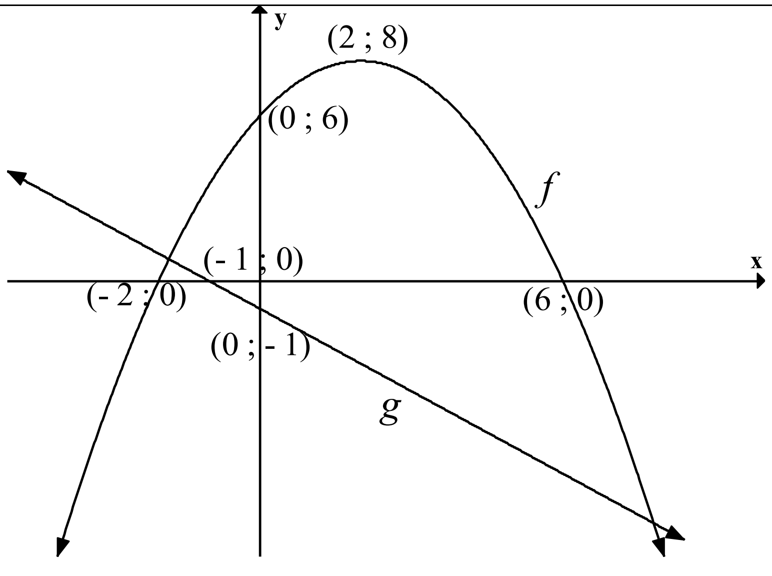
3.1	$a + ar = 2$ $a(1+r) = 2$ $a = \frac{2}{1+r}$	✓ $a + ar = 2$ ✓ $a = \frac{2}{1+r}$ (2)
	OR/*OF* $\frac{a}{1-r} - 2 = \frac{1}{4}$ $4a - 8(1-r) = 1 - r$ $4a - 8 + 8r = 1 - r$ $4a = 9 - 9r$ $a = \frac{9-9r}{4}$	✓ $\frac{a}{1-r} - 2 = \frac{1}{4}$ ✓ $a = \frac{9-9r}{4}$ (2)
	OR/*OF* $S_n = \frac{a(r^n - 1)}{r-1}$ $2 = \frac{a(r^2-1)}{r-1}$ $2 = \frac{a(r-1)(r+1)}{r-1}$ $2 = a(r+1)$ $a = \frac{2}{r+1}$	**OR/*OF*** ✓ $2 = \frac{a(r^2-1)}{r-1}$ ✓ $a = \frac{2}{1+r}$ (2)
	OR/*OF* $\frac{ar^2}{1-r} = \frac{1}{4}$ $a = \frac{1-r}{4r^2}$	**OR/*OF*** ✓ $\frac{ar^2}{1-r} = \frac{1}{4}$ ✓ $a = \frac{1-r}{4r^2}$ (2)

3.2	$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$ $S_\infty = 2 + \frac{1}{4}$ $\frac{a}{1-r} = 2 + \frac{1}{4}$ $\frac{a}{1-r} = \frac{9}{4}$ $\left(\frac{2}{1+r}\right)\times\left(\frac{1}{1-r}\right) = \frac{9}{4}$ $\frac{2}{1-r^2} = \frac{9}{4}$ $8 = 9 - 9r^2$ $9r^2 = 1$ $r = \frac{1}{3}$ $a = \frac{3}{2}$	✓ $S_\infty = 2 + \frac{1}{4}$ ✓ $\frac{a}{1-r} = \frac{9}{4}$ ✓ substitution of a into the correct formula ✓ $9r^2 = 1$ ✓ $r = \frac{1}{3}$ ✓ $a = \frac{3}{2}$ (6)
	OR/*OF* $S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$ $S_\infty = 2 + \frac{1}{4}$ $\frac{a}{1-r} = 2 + \frac{1}{4}$ $\frac{a}{1-r} = \frac{9}{4}$ $4a = 9 - 9r$ $r = \frac{9-4a}{9}$ $a + a\left(\frac{9-4a}{9}\right) = 2$ $9a + 9a - 4a^2 = 18$ $2a^2 - 9a + 9 = 0$ $(a-3)(2a-3) = 0$ $a = \frac{3}{2}$ or $a = 3$ $r = \frac{1}{3}$ or $r = -\frac{1}{3}$ N/A	**OR/*OF*** ✓ $S_\infty = 2 + \frac{1}{4}$ ✓ $\frac{a}{1-r} = \frac{9}{4}$ ✓ $r = \frac{9-4a}{9}$ ✓ substitution of a into the correct formula ✓ $a = \frac{3}{2}$

	OR/*OF* $r=\frac{2-a}{a}$ $\frac{ar^2}{1-r}=\frac{1}{4}$ $4ar^2=1-r$ $4a\left(\frac{2-a}{a}\right)^2=1-\frac{2-a}{a}$ $16-16a+4a^2=2a+2$ $2a^2-9a+9=0$ $(2a-3)(a-3)=0$ $a=\frac{3}{2}$ $\quad a\neq 3$ $r=\frac{1}{3}$ $\quad r\neq -\frac{1}{3}$	✓ $r=\frac{1}{3}$ (6) **OR/*OF*** ✓ $r=\frac{2-a}{a}$ ✓ $\frac{ar^2}{1-r}=\frac{1}{4}$ ✓ substitution of *a* ✓ $(2a-3)(a-3)=0$ ✓ $a=\frac{3}{2}$ ✓ $r=\frac{1}{3}$ (6)
	OR/*OF* $S_\infty=T_1+T_2+\sum_{n=3}^{\infty}T_n$ $S_\infty=2+\frac{1}{4}$ $\frac{a}{1-r}=2+\frac{1}{4}$ $\frac{a}{1-r}=\frac{9}{4}$ $\left(\frac{1-r}{4r^2}\right)\times\left(\frac{1}{1-r}\right)=\frac{9}{4}$ $\frac{1}{4r^2}=\frac{9}{4}$ $4=36r^2$ $9r^2=1$ $r=\frac{1}{3}$ $a=\frac{3}{2}$	**OR/*OF*** ✓ $S_\infty=2+\frac{1}{4}$ ✓ $\frac{a}{1-r}=\frac{9}{4}$ ✓ substitution of *a* ✓ $9r^2=1$ ✓ $r=\frac{1}{3}$ ✓ $a=\frac{3}{2}$ (6) **[8]**

QUESTION/*VRAAG* 4

4.1	$f(x)=-ax^2+bx+6$ $f^{/}(x)=-2ax+b$ $-2ax+b=3$ at $x=-1$ $2a+b=3 \quad [1]$ $f(-1)=\frac{7}{2}$ $-a-b+6=\frac{7}{2}$ $-2a-2b+12=7$ $2a+2b=5 \quad [2]$ $[2]-[1]$ $b=2$ $2a+2=3$ $a=\frac{1}{2}$	✓$-2ax+b$ ✓✓ $2a+b=3$ ✓ $-a-b+6=\frac{7}{2}$ ✓solve simultaneously (5)
	OR/*OF* $f^{/}(x)=-2ax+b$ $3=2a+b$ $b=3-2a$ $\frac{7}{2}=-a(-1)^2+(3-2a)(-1)+6$ $a+3=\frac{7}{2}$ $a=\frac{1}{2}$ $b=2$	✓$-2ax+b$ ✓✓ $2a+b=3$ ✓ $\frac{7}{2}=-a(-1)^2+(3-2a)(-1)+6$ ✓solve simultaneously (5)
4.2	$f(x)=-\frac{1}{2}x^2+2x+6$ $x-$intercepts: $-\frac{1}{2}x^2+2x+6=0$ $-x^2+4x+12=0$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $(-2\,;0) \quad (6\,;0)$	✓ $-\frac{1}{2}x^2+2x+6=0$ ✓$(-2\,;0)$ ✓ $(6\,;0)$ (3)

4.3	$f(x)=-\frac{1}{2}x^2+2x+6$ $f'(x)=0$ or $x=-\frac{b}{2a}$ or $x=\frac{-2+6}{2}$ $-x+2=0$ $x=-\frac{2}{2.\left(-\frac{1}{2}\right)}$ $x=2$ $x=2$ $x=2$ $y=-\frac{1}{2}(2)^2+2(2)+6$ $=-2+4+6$ $=8$ TP $(2\,;8)$ **OR/OF** $y=-\frac{1}{2}\left(x^2-4x-12\right)$ $=-\frac{1}{2}\left[(x-2)^2-4-12\right]$ $=-\frac{1}{2}(x-2)^2+8$ TP $(2\,;8)$	✓ $-x+2$ / $-\frac{2}{2.\left(-\frac{1}{2}\right)}$ / $\frac{-2+6}{2}$ ✓ $x=2$ ✓ $y=8$ **OR/*OF*** ✓ $-\frac{1}{2}(x-2)^2+8$ ✓ $x=2$ ✓ $y=8$ (3)
4.4 4.6		4.4: *f*: ✓shape ✓*x*- intercepts ✓*y*- intercept ✓(2 ; 8) (4) 4.6: *g*: ✓ *x*- intercept ✓ *y*- intercept (2)
4.5	$0<x<4$ or $(0\,;4)$	✓ 4 ✓✓ $0<x<4$ (3)
4.7	$x\le -2$ or $-1\le x\le 6$ **OR/OF** $(-\infty\,;-2]$ or $[-1;6]$	✓ $x\le -2$ ✓✓ $-1\le x\le 6$ (3) **[23]**

QUESTION/*VRAAG* 5

5.1	$y \in R\,;\, y \neq -1$ **OR/*OF*** $y < -1$ or $y > -1$ **OR/*OF*** $y \in (-\infty\,;-1)$ or $y \in (-1\,;\infty)$ **OR/*OF*** $R - \{-1\}$	✓✓ answer (2)
5.2	$D(2\,;-1)$ $g(x) = \frac{2}{x-2} - 1$	✓ $D(2\,;-1)$ ✓ $\frac{2}{x-2} - 1$ (2)
5.3	$f(x) = \log_3 x.$ $\log_3 t = 1$ $t = 3$ **OR/*OF*** $g(x) = \frac{2}{x-2} - 1$ $1 = \frac{2}{t-2} - 1$ $2 = \frac{2}{t-2}$ $2t - 4 = 2$ $t = 3$	✓correct substitution of A ✓✓ $t = 3$ (3)
5.4	$x = \log_3 y$ $y = 3^x$	✓interchange x and y ✓ $y = 3^x$ (2)
5.5	$3^x < 3^1$ $x < 1$ **OR/*OF*** $3^x < 3^1$ $x \in (-\infty\,;1)$	✓ $3^x < 3^1$ ✓ $x < 1$ (2) ✓ $3^x < 3^1$ ✓ $x \in (-\infty\,;1)$ (2)
5.6	Equation of the axis of symmetry: $y = -x + 1$ x-intercept of the axis of symmetry is at $x = 1$ f has an x-intercept at B(1 ; 0) which is the same as the x-intercept of the axis of symmetry Point of intersection: B $(1\,;0)$ **OR/*OF*** Since BE = ED = 1 and D lies on the axis of symmetry and the gradient of the axis of symmetry is –1, B will also lie on the axis of symmetry. But B also lies on f. Therefore B(1 ; 0) is the point of intersection between f and the axis of symmetry with a negative gradient./ *Omdat* BE = ED = 1 *en* D *op die simmetrie-as lê en die simmetrie-as se gradiënt* –1 *is, sal* B *ook op die simmetrie-as lê. Maar* B *lê ook op f. Dus is* B(1 ; 0) *die snypunt van f en die simmetrie-as met negatiewe gradiënt.*	✓✓ equation of axis of symmetry ✓B or $(1\,;0)$ **OR/*OF*** ✓✓ BE = ED = 1 ✓ B or $(1\,;0)$ (3) **[14]**

QUESTION/*VRAAG* 6

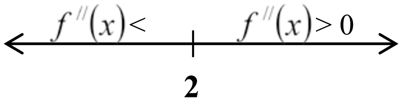
6.1	$A = P(1+i)^n$ $12\ 146{,}72 = 10\ 000\left(1+\frac{r}{12}\right)^{36}$ $\left(1+\frac{r}{12}\right)^{36} = 1{,}214672$ $1+\frac{r}{12} = \sqrt[36]{1{,}214672}$ $= 1{,}005416$ $\frac{r}{12} = 0{,}005416$ $r = 0{,}06500$ $r = 6{,}5\%$	✓ $\frac{r}{12}$ ✓ $n = 36$ ✓ correct substitution into formula ✓ $1+\frac{r}{12} = \sqrt[36]{1{,}214672}$ ✓ 6,5% (5)
6.2.1	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $235\ 000 = \frac{x\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}{\frac{0{,}11}{12}}$ $x = \frac{235\ 000\times\frac{0{,}11}{12}}{\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}$ $= R5\ 536{,}95$ His monthly instalment is R 5 536,95	✓ $i = \frac{0{,}11}{12}$ ✓ $n = 54$ ✓ correct substitution in P ✓ answer (4)
6.2.2	Amount paid for the year : $(5\ 536{,}95\times 12) =$ R66 443,40 $\text{Balance} = 235\ 000\left(1+\frac{0{,}11}{12}\right)^{12} - \frac{5\ 536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12}-1\right]}{\frac{0{,}11}{12}}$ $= 192\ 296{,}17$ $\text{Interest} = (5\ 536{,}95\times 12)-(235\ 000-192\ 296{,}17)$ $= 66\ 443{,}40 - 42\ 703{,}83$ $= 23\ 739{,}57$ **OR/*OF***	✓ R66 443,40 ✓ $235\ 000\left(1+\frac{0{,}11}{12}\right)^{12}$ ✓ $\frac{5\ 536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12}-1\right]}{\frac{0{,}11}{12}}$ ✓ R192 296,17 ✓ R42 703,83 ✓ R23 739,57 **OR/*OF***

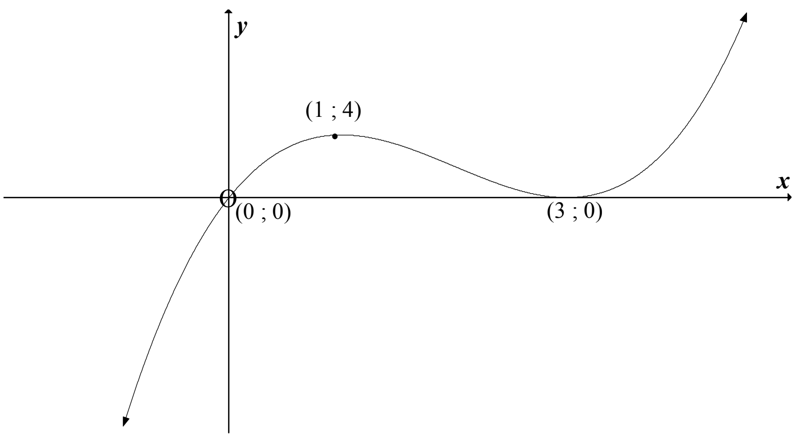
	Total amount paid in first year $= \text{R } 5\ 536.95 \times 12$ $= \text{R}66\ 443{,}40$ Balance on loan after 1 year = P of remaining installments $\text{P} = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ $= \dfrac{5\ 536{,}95\left[1-\left(1+\frac{0{,}11}{12}\right)^{-42}\right]}{\frac{0{,}11}{12}}$ $= \text{R}192\ 296{,}20$ Amount paid off in the first year: R235 000 – R192 296,20 = R42 703,80 Amount of interest = R66 443,40 – R42 703,80 = R23 739,60 **OR/*OF*** $P = \dfrac{5536{,}95\left[1-\left(1+\frac{0{,}11}{12}\right)^{-12}\right]}{\frac{0{,}11}{12}}$ $= R\ 62\ 648{,}18$ 235 000 – 62 648,18 = R172 351,82 After 12 months, money owed on house is $172\ 351{,}82\left(1+\frac{0{,}11}{12}\right)^{12}$ $= 192\ 296{,}17$ Amount paid after 12 months is 5 536,95 × 12 = R 66 443, 40 Amount of interest paid: R 66 443, 40 – (235 000 – 192 296,17) = R 23 739, 57	✓ R66 443,40 ✓ $n = -42$ ✓ substitution into correct formula ✓ R192 296,20 ✓ R42 703,80 ✓ R23 739,60 (6) **OR/*OF*** ✓ R62 648,18 ✓ R172 351,82 ✓ R192 296,17 ✓ R66 443,40 ✓ 235 000 – 192 296,17 ✓ R23 739,57 (6) **[15]**

QUESTION/*VRAAG* 7

7.1	$f(x+h)=2(x+h)^2-(x+h)$ $=2(x^2+2xh+h^2)-x-h$ $=2x^2+4xh+2h^2-x-h$ $f(x+h)-f(x)=2x^2+4xh+2h^2-x-h-2x^2+x$ $=4xh+2h^2-h$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2-h}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h-1)}{h}$ $=\lim_{h\to 0}(4x+2h-1)$ $=4x-1$ **OR/*OF*** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2(x+h)^2-(x+h)-(2x^2-x)}{h}$ $=\lim_{h\to 0}\frac{2x^2+4xh+2h^2-x-h-2x^2+x}{h}$ $=\lim_{h\to 0}\frac{4xh+2h^2-h}{h}$ $=\lim_{h\to 0}\frac{h(4x+2h-1)}{h}$ $=\lim_{h\to 0}(4x+2h-1)$ $=4x-1$	✓ $2x^2+4xh+2h^2-x-h$ ✓ $4xh+2h^2-h$ ✓ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ✓subst. into formula ✓ $\lim_{h\to 0}(4x+2h-1)$ ✓ $4x-1$ **OR/*OF*** ✓ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ✓subst. into formula ✓ $2x^2+4xh+2h^2-x-h$ ✓ $4xh+2h^2-h$ ✓ $\lim_{h\to 0}(4x+2h-1)$ ✓ $4x-1$ (6)
7.2.1	$D_x[(x+1)(3x-7)]$ $=D_x(3x^2-4x-7)$ $=6x-4$	✓ $3x^2-4x-7$ ✓ $6x-4$ (2)
7.2.2	$y=\sqrt{x^3}-\frac{5}{x}+\frac{1}{2}\pi$ $y=x^{\frac{3}{2}}-5x^{-1}+\frac{1}{2}\pi$ $\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}+5x^{-2}$	✓ $x^{\frac{3}{2}}-5x^{-1}$ ✓ $\frac{3}{2}x^{\frac{1}{2}}$ ✓ $+5x^{-2}$ ✓ derivative of $\frac{1}{2}\pi$ is 0 (4) **[12]**

QUESTION/*VRAAG* 8

8.1	$f(x)=x^3-6x^2+9x$
	$f^{/}(x)=3x^2-12x+9$
	$f^{//}(x)=6x-12=0$
	$x=2$
	$f^{//}(0)=6(0)-12$
	$=-12$
	$f^{//}(3)=6(3)-12$
	$=6$
	
	Point of inflection at $x=2$
		✓ x^3-6x^2+9x
		✓ $3x^2-12x+9$
		✓ $6x-12$
		✓ $6x-12=0$
		✓ explanation
		(5)

8.2		✓shape ✓ (0 ; 0) ✓ (3 ; 0) as TP ✓(1 ; 4) (4)
8.3	f concave up for $x>2$ $y=-f(x)$ will be concave down for $x>2$	✓✓ $x>2$ (2)
8.4.1	$(3;7)$	✓ 3 ✓7 (2)
8.4.2	Do not agree with Claire as her statement is incorrect. Between $x = 1$ and $x = 3$ the graph of f is decreasing. Therefore at $x = 2$ the gradient will have a negative value. *Stem nie saam met Claire nie, want haar stelling in verkeerd.* *Die grafiek van f is dalend/afnemend tussen* $x = 1$ en $x = 3$. *By $x = 2$ moet die gradiënt dus 'n negatiewe waarde hê.* **OR/*OF*** $f^{/}(2)=3(2)^2-12(2)+9$ $=-3$ $\neq 1$	✓no ✓justification (2) **[15]**

QUESTION/*VRAAG* 9

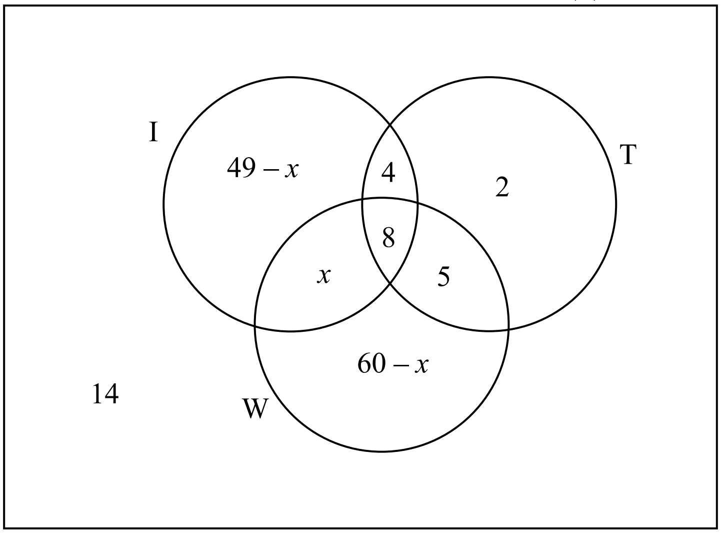
	$y = x^2 + 2$ $P(x\,;x^2+2)$ $B(0\,;3)$ $PB^2 = (x-0)^2 + (x^2+2-3)^2$ $= x^2 + x^4 - 2x^2 + 1$ $= x^4 - x^2 + 1$ PB will be a minimum if PB^2 is a minimum $\frac{d(PB^2)}{dx} = 4x^3 - 2x$ $4x^3 - 2x = 0$ $x(2x^2-1) = 0$ $x = 0$ or $x^2 = \frac{1}{2}$ $x = \frac{1}{\sqrt{2}}$ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $= \frac{1}{4} - \frac{1}{2} + 1$ $= \frac{3}{4}$ $PB = \frac{\sqrt{3}}{2} = 0{,}87$ **OR/*OF***	✓ $(x-0)^2 + (x^2+2-3)^2$ ✓ $x^4 - x^2 + 1$ ✓ $4x^3 - 2x$ ✓ $\frac{d(PB^2)}{dx} = 0$ ✓ $x = \frac{1}{\sqrt{2}}$ ✓ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ ✓ answer **OR/*OF***

	Gradient of tangent to curve $=2x$	✓ $=2x$
	Gradient of line joining B and the curve $=\dfrac{x^2+2-3}{x-0}$	
	$=\dfrac{x^2-1}{x}$	✓ $=\dfrac{x^2-1}{x}$
	Shortest distance will be where tangent to curve is perpendicular to the line joining P and the curve.	
	$\dfrac{x^2-1}{x}=-\dfrac{1}{2x}$	✓ $\dfrac{x^2-1}{x}=-\dfrac{1}{2x}$
	$2x(x^2-1)=-x$	
	$2x^3-2x=0$	✓ $2x^3-2x=0$
	$x(2x^2-1)=0$	
	$x=0$ or $x^2=\dfrac{1}{2}$	
	$x=\dfrac{1}{\sqrt{2}}$	✓ $x=\dfrac{1}{\sqrt{2}}$
	$PB^2=\left(\dfrac{1}{\sqrt{2}}\right)^4-\left(\dfrac{1}{\sqrt{2}}\right)^2+1$	✓ $PB^2=\left(\dfrac{1}{\sqrt{2}}\right)^4-\left(\dfrac{1}{\sqrt{2}}\right)^2+1$
	$=\dfrac{1}{4}-\dfrac{1}{2}+1$	
	$=\dfrac{3}{4}$	
	$PB=\dfrac{\sqrt{3}}{2}=0,87$	✓ answer
	OR/*OF*	**OR/*OF***
	$P(k;k^2+2)$ and B(0; 3)	✓ $P(k;k^2+2)$
	$BP\perp$ tangent passing through $y=x^2+2$ at P.	
	$m_{\text{tangent at P}}=2k$	✓ $m_{\text{tangent at P}}=2k$
	$m_{BP}=-\dfrac{1}{2k}$	✓ $m_{BP}=-\dfrac{1}{2k}$
	Equation of BP: $y=\left(-\dfrac{1}{2k}\right)x+3$	✓ $y=\left(-\dfrac{1}{2k}\right)x+3$
	$y_P=\left(-\dfrac{1}{2k}\right)(k)+3=2,5$	✓ value of y at P
	$\Rightarrow k^2+2=2,5$ and so $k=\sqrt{0,5}$ and $P\left(\sqrt{0,5};\ 2,5\right)$	✓value of k
	$BP=\sqrt{\left(\sqrt{0,5}-0\right)^2+(2,5-3)^2}=\sqrt{\dfrac{3}{4}}=\dfrac{\sqrt{3}}{2}=0,87$	✓ answer
		[7]

QUESTION/*VRAAG* 10

10.1

n(S) = 100



8 values need to be placed in correct position:

2 or 3 correct: 1 mark
4 or 5 correct: 2 marks
6 or 7 correct: 3 marks
8 correct: 4 marks

(4)

10.2

$$(49 - x) + x + 8 + 4 + 5 + 2 + (60 - x) + 14 = 100$$
$$-x + 142 = 100$$
$$x = 42$$

✓ setting up equation
✓ answer
(2)

10.3

$$\text{P (use only one application)} = \frac{7+2+18}{100} = \frac{27}{100} \text{ or } 27\%$$

✓ $\frac{7+2+18}{100}$
✓answer
(2)
[8]

QUESTION/*VRAAG* 11

11.1	5 x 5 x 10 x 9 = 2250	✓ 5 x 5 ✓ 10 x 9 ✓ 2250 (3)

11.2

No of digits used	Letters	Digits	Total
1	5 x 5	10	250
2	5 x 5	10 x 9	2 250
3	5 x 5	10 x 9 x 8	18 000
4	5 x 5	10 x 9 x 8 x7	126 000
5	5 x 5	10 x 9 x 8 x7 x 6	756 000

Codes of two letters and five digits will ensure unique numbers for 700 000 clients.

✓ 5 x 5 x 10 x 9 x 8 x 7 x 6
✓✓ five digits (3)
[6]

TOTAL/*TOTAAL*: 150

MATHEMATICS P1

2018

QUESTION PAPER

Inside Education
Inspiring Minds

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 4x + 3 = 0$ (3)

1.1.2 $5x^2 - 5x + 1 = 0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 - 3x - 10 > 0$ (3)

1.1.4 $3\sqrt{x} = x - 4$ (4)

1.2 Solve simultaneously for x and y:

$3x - y = 2$ and $2y + 9x^2 = -1$ (6)

1.3 If $3^{9x} = 64$ and $5^{\sqrt{p}} = 64$, calculate, WITHOUT the use of a calculator, the value of: $\dfrac{\left[3^{x-1}\right]^3}{\sqrt{5}^{\sqrt{p}}}$ (4)

[23]

QUESTION 2

2.1 Given the quadratic sequence: 2 ; 3 ; 10 ; 23 ; …

2.1.1 Write down the next term of the sequence. (1)

2.1.2 Determine the n^{th} term of the sequence. (4)

2.1.3 Calculate the 20^{th} term of the sequence. (2)

2.2 Given the arithmetic sequence: 35 ; 28 ; 21 ; …

Calculate which term of the sequence will have a value of –140. (3)

2.3 For which value of n will the sum of the first n terms of the arithmetic sequence in QUESTION 2.2 be equal to the n^{th} term of the quadratic sequence in QUESTION 2.1? (6)

[16]

QUESTION 3

A geometric series has a constant ratio of $\frac{1}{2}$ and a sum to infinity of 6.

3.1 Calculate the first term of the series. (2)

3.2 Calculate the 8^{th} term of the series. (2)

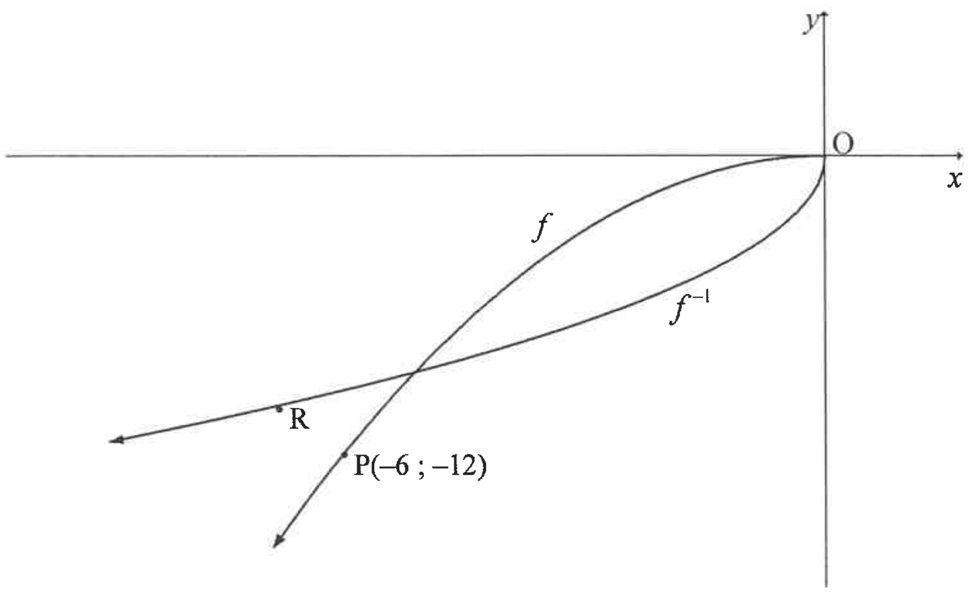
3.3 Given: $\sum_{k=1}^{n} 3(2)^{1-k} = 5{,}8125$ Calculate the value of n. (4)

3.4 If $\sum_{k=1}^{20} 3(2)^{1-k} = p$, write down $\sum_{k=1}^{20} 24(2)^{-k}$ in terms of p. (3)

[11]

QUESTION 4

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \le 0$.
The graph of f^{-1} is also drawn. P(–6 ; –12) is a point on f and R is a point on f^{-1}.



4.1 Is f^{-1} a function? Motivate your answer. (2)

4.2 If R is the reflection of P in the line $y = x$, write down the coordinates of R. (1)

4.3 Calculate the value of a. (2)

4.4 Write down the equation of f^{-1} in the form $y = ...$ (3)

[8]

QUESTION 5

Given: $f(x) = \dfrac{-1}{x-1}$

5.1 Write down the domain of f. (1)

5.2 Write down the asymptotes of f. (2)

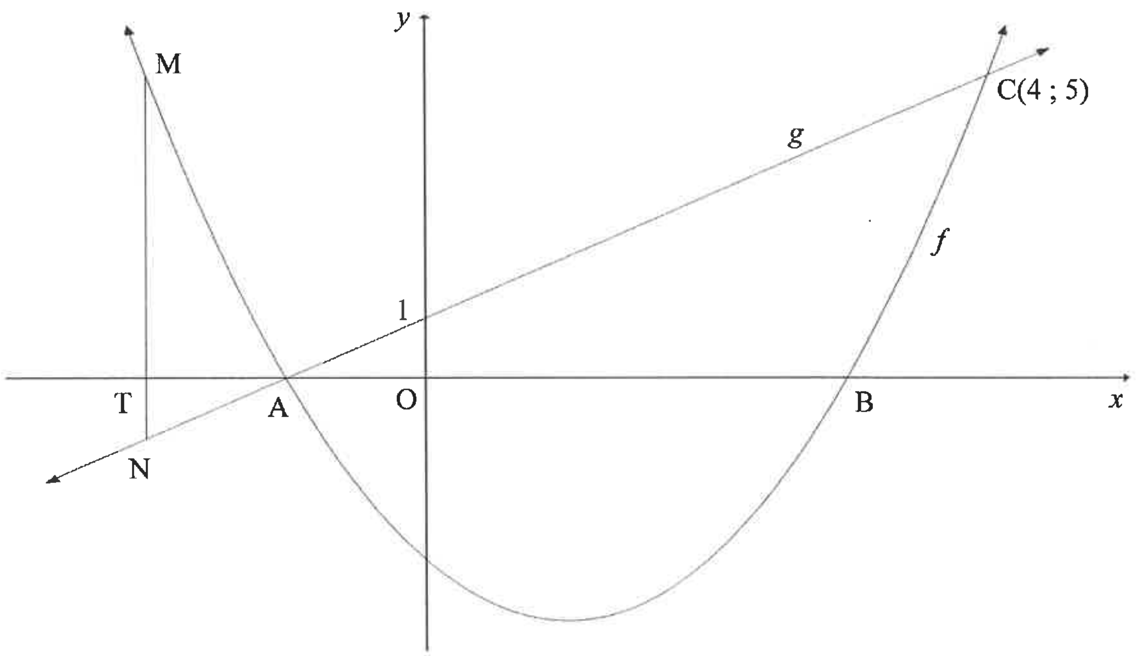
5.3 Sketch the graph of f, clearly showing all intercepts with the axes and any asymptotes. (3)

5.4 For which values of x will $x.f'(x) \geq 0$? (2)

[8]

QUESTION 6

In the diagram below, A and B are the x-intercepts of the graph of $f(x) = x^2 - 2x - 3$.
A straight line, g, through A cuts f at C(4 ; 5) and the y-axis at (0 ; 1).
M is a point on f and N is a point on g such that MN is parallel to the y-axis.
MN cuts the x-axis at T.



6.1 Show that $g(x) = x + 1$. (2)

6.2 Calculate the coordinates of A and B. (3)

6.3 Determine the range of f. (3)

6.4 If MN = 6:

6.4.1 Determine the length of OT if T lies on the negative x-axis. Show ALL your working. (4)

6.4.2 Hence, write down the coordinates of N. (2)

6.5 Determine the equation of the tangent to f drawn parallel to g. (5)

6.6 For which value(s) of k will $f(x) = x^2 - 2x - 3$ and $h(x) = x + k$ NOT intersect? (1)

[20]

QUESTION 7

7.1 Selby decided today that he will save R15 000 per quarter over the next four years. He will make the first deposit into a savings account in three months' time and he will make his last deposit at the end of four years from now.

7.1.1 How much will Selby have at the end of four years if interest is earned at 8,8% per annum, compounded quarterly? (3)

7.1.2 If Selby decides to withdraw R100 000 from the account at the end of three years from now, how much will he have in the account at the end of four years from now? (3)

7.2 Tshepo takes out a home loan over 20 years to buy a house that costs R1 500 000.

7.2.1 Calculate the monthly instalment if interest is charged at 10,5% p.a., compounded monthly. (4)

7.2.2 Calculate the outstanding balance immediately after the 144^{th} payment was made. (5)

[15]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given $f(x) = x^2 - 5$. (5)

8.2 Determine $\frac{dy}{dx}$ if:

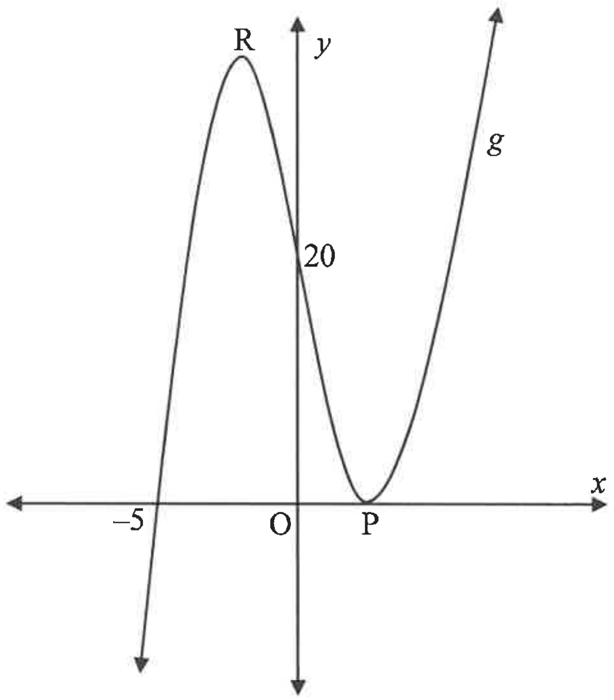
8.2.1 $y = 3x^3 + 6x^2 + x - 4$ (3)

8.2.2 $yx - y = 2x^2 - 2x \quad ; \quad x \neq 1$ (4)

[12]

QUESTION 9

9.1 The graph of $g(x) = x^3 + bx^2 + cx + d$ is sketched below.
The graph of g intersects the x-axis at $(-5\ ;\ 0)$ and at P, and the y-axis at $(0\ ;\ 20)$.
P and R are turning points of g.



9.1.1 Show that $b = 1$, $c = -16$ and $d = 20$. (4)

9.1.2 Calculate the coordinates of P and R. (5)

9.1.3 Is the graph concave up or concave down at $(0\ ;\ 20)$? Show ALL your calculations. (3)

9.2 If g is a cubic function with:

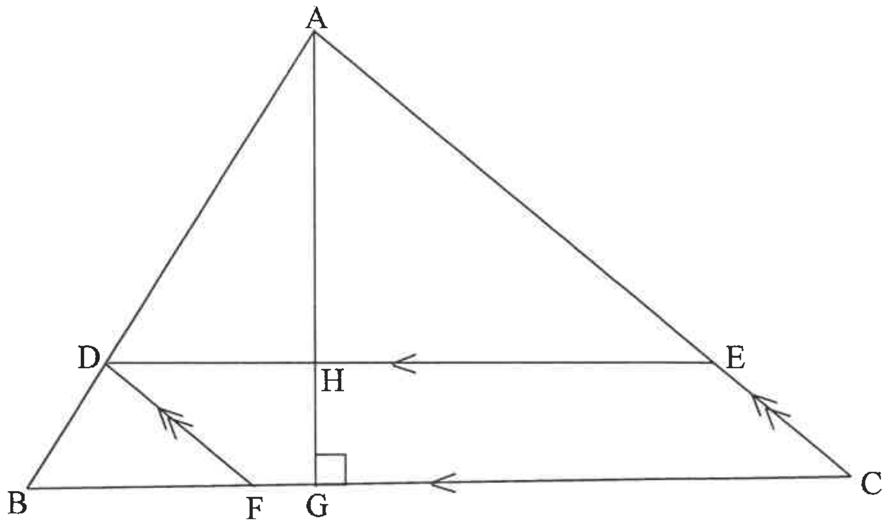
- $g(3) = g'(3) = 0$
- $g(0) = 27$
- $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,

draw a sketch graph of g indicating ALL relevant points. (3)
[15]

QUESTION 10

In ΔABC:

- D is a point on AB, E is a point on AC and F is a point on BC such that DECF is a parallelogram.
- BF : FC = 2 : 3.
- The perpendicular height AG is drawn intersecting DE at H.
- AG = t units.
- BC = $(5 - t)$ units.



10.1 Write down AH : HG. (1)

10.2 Calculate t if the area of the parallelogram is a maximum.
(**NOTE:** Area of a parallelogram = base × $\perp$ height) (5)
[6]

QUESTION 11

Given the digits: 3 ; 4 ; 5 ; 6 ; 7 ; 8 and 9

11.1 Calculate how many unique 5-digit codes can be formed using the digits above, if:

11.1.1 The digits may be repeated (2)

11.1.2 The digits may not be repeated (2)

11.2 How many unique 3-digit codes can be formed using the above digits, if:

- Digits may be repeated
- The code is greater than 400 but less than 600
- The code is divisible by 5 (3)

[7]

QUESTION 12

12.1 Given: P(A) = 0,45; P(B) = y and P(A or B) = 0,74

Determine the value(s) of y if A and B are mutually exclusive. (3)

12.2 An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a mystery gift in exactly $\frac{1}{4}$ of the total number of bags.

Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects two bags of sweets with a mystery gift is $\frac{7}{118}$. Calculate the number of gift bags of sweets with a mystery gift inside. (6)

[9]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

MATHEMATICS P1

2018

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NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD* 12

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS: 150
PUNTE: 150

These marking guidelines consist of 18 pages.
Hierdie nasienriglyne bestaan uit* 18 *bladsye

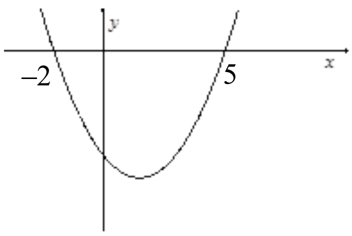
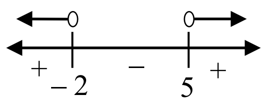
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION/*VRAAG* 1

1.1.1	$x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=3$ or $x=1$	✓factors/correct subt in formula ✓ $x=3$ ✓ $x=1$ (3)
1.1.2	$5x^2-5x+1=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\dfrac{5\pm\sqrt{25-4(5)(1)}}{2(5)}$ $=\dfrac{5\pm\sqrt{5}}{10}$ $x=0,72$ or $x=0,28$	✓substitution into the correct formula ✓$x=0,72$ ✓$x=0,28$ (3)
1.1.3	$x^2-3x-10>0$ $(x-5)(x+2)>0$ **OR/OF**  $x<-2$ or $x>5$	✓factors/ critical values ✓✓ $x<-2$ or $x>5$ (3)
1.1.4	$3\sqrt{x}=x-4$ $9x=x^2-8x+16$ $x^2-17x+16=0$ $(x-16)(x-1)=0$ $x=16$ or $x=1$ NA	✓ squaring both sides ✓ $x^2-17x+16=0$ ✓ factors ✓ answer with selection (4)

	OR/OF $3x^{\frac{1}{2}} = x - 4$ $x - 3x^{\frac{1}{2}} - 4 = 0$ $\left(x^{\frac{1}{2}} - 4\right)\left(x^{\frac{1}{2}} + 1\right) = 0$ $x^{\frac{1}{2}} = 4$ or $x^{\frac{1}{2}} = -1$ $x = 16$ NA	**OR/OF** ✓ standard form ✓ recognize $x = \left(x^{\frac{1}{2}}\right)^2$ ✓ factors ✓ answer with selection (4)
1.2	$2y + 9x^2 = -1$......(1) $3x - y = 2$ ...... (2) $y = 3x - 2$ ....(3) $2(3x-2) + 9x^2 = -1$ $6x - 4 + 9x^2 = -1$ $9x^2 + 6x - 3 = 0$ $3x^2 + 2x - 1 = 0$ $(3x-1)(x+1) = 0$ $x = \frac{1}{3}$ or $x = -1$ $y = -1$ or $y = -5$	✓ $y = 3x - 2$ ✓substitution ✓ standard form ✓factors ✓both x values ✓both y values (6)
	OR/OF $2y + 9x^2 = -1$......(1) $3x - y = 2$ ...... (2) $x = \frac{y+2}{3}$ $2y + 9\left(\frac{y+2}{3}\right)^2 = -1$ $2y + 9\left(\frac{y^2 + 4y + 4}{9}\right) = -1$ $2y + y^2 + 4y + 4 + 1 = 0$ $y^2 + 6y + 5 = 0$ $(y+5)(y+1) = 0$ $y = -1$ or $y = -5$ $x = \frac{1}{3}$ or $x = -1$	**OR/OF** ✓ $x = \frac{y+2}{3}$ ✓substitution ✓ standard form ✓factors ✓both y values ✓both x values (6)

1.3	$3^{9x} = 64$ $\left(3^{3x}\right)^3 = (4)^3$ $3^{3x} = 4$ $5^{\sqrt{p}} = 64$ $\sqrt{5}^{\sqrt{p}} = \sqrt{64}$ $\sqrt{5}^{\sqrt{p}} = 8$ $\frac{\left[3^{x-1}\right]^3}{\sqrt{5}^{\sqrt{p}}} = \frac{3^{3x-3}}{\sqrt{5}^{\sqrt{p}}}$ **OR/OF** $= \frac{3^{3x}.3^{-3}}{5^{\frac{\sqrt{p}}{2}}}$ $= \frac{3^{3x}}{27 \times \sqrt{5}^{\sqrt{p}}}$ $= \frac{\sqrt[3]{64}.3^{-3}}{\sqrt{64}}$ $= \frac{4}{27 \times 8}$ $= \frac{1}{54}$	✓ $3^{3x} = 4$ ✓ $\sqrt{5}^{\sqrt{p}} = 8$ ✓ 3^{3x-3} or $3^{3x}.3^{-3}$ ✓answer (4)
	OR/OF $\frac{\left(3^{x-1}\right)^3}{\sqrt{5}^{\sqrt{p}}} = \frac{3^{3x}.3^{-3}}{\left(5^{0.5}\right)^{\sqrt{p}}}$ $= \frac{3^{3x}.3^{-3}}{\left(5^{\sqrt{p}}\right)^{0.5}}$ $= \frac{4.3^{-3}}{\sqrt{64}}$ $= \frac{4.\frac{1}{27}}{8} = \frac{1}{54}$	**OR/OF** ✓ 3^{3x-3} or $3^{3x}.3^{-3}$ ✓ $3^{3x} = 4$ ✓ $\sqrt{5}^{\sqrt{p}} = 8$ ✓answer (4)
		[23]

QUESTION/*VRAAG* 2

2.1.1	42	✓answer (1)
2.1.2	$2a = 6$ $3a + b = 1$ $a + b + c = 2$ $a = 3$ $3(3) + b = 1$ $(3) + (-8) + c = 2$ $b = -8$ $c = 7$ $T_n = 3n^2 - 8n + 7$ **OR/OF** $2a = 6$ $a = 3$ $T_n = 3n^2 + bn + c$ $T_1 : 3 + b + c = 2$ $b + c = -1$ ......(1) $T_2 : 12 + 2b + c = 3$ $2b + c = -9$ ......(2) $T_2 - T_1 : b = -8$ Subst. in (1): $-8 + c = -1$ $c = 7$ $T_n = 3n^2 - 8n + 7$	✓ $a = 3$ ✓ $b = -8$ ✓ $c = 7$ ✓ $T_n = an^2 + bn + c$ (4) **OR/OF** ✓ $a = 3$ ✓ $b = -8$ ✓ $c = 7$ ✓ $T_n = an^2 + bn + c$ (4)
2.1.3	$T_{20} = 3(20)^2 - 8(20) + 7$ $= 1047$	✓substitution ✓answer (2)
2.2	$T_n = -7n + 42$ $-7n + 42 = -140$ $-7n = -182$ $n = 26$	✓ $T_n = -7n + 42$ ✓ $-7n + 42 = -140$ ✓ $n = 26$ (3)
2.3	$S_n = \frac{n}{2}(a + l)$ **OR/OF** $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{n}{2}(35 - 7n + 42)$ $S_n = \frac{n}{2}(70 - 7n + 7)$ $S_n = \frac{n}{2}(-7n + 77)$ $S_n = -\frac{7}{2}n^2 + \frac{77}{2}n$ $-\frac{7}{2}n^2 + \frac{77}{2}n = 3n^2 - 8n + 7$ $13n^2 - 93n + 14 = 0$ $(n-7)(13n-2) = 0$ $n = 7$ *or* $n = \frac{2}{13}$ NA $\therefore n = 7$	 ✓ $S_n = \frac{n}{2}(35 - 7n + 42)$ or $S_n = \frac{n}{2}(70 - 7n + 7)$ ✓simplification of S_n ✓equating ✓standard form ✓factors ✓answer with selection (6)
		[16]

QUESTION/*VRAAG* 3

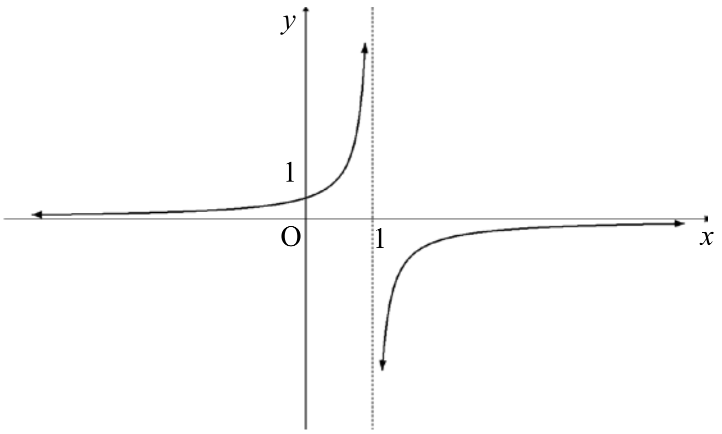
3.1	$r=\frac{1}{2}$ and $S_\infty=6$ $S_\infty=\frac{a}{1-r}$ $6=\frac{a}{1-\frac{1}{2}}$ $a=3$	✓substitution ✓answer (2)
3.2	$T_n=ar^{n-1}$ $T_8=3\left(\frac{1}{2}\right)^7$ $T_8=\frac{3}{128}$	✓✓ $T_8=3\left(\frac{1}{2}\right)^7$ (2)
3.3	$\sum_{k=1}^{n}3(2)^{1-k}=5,8125$ $3+\frac{3}{2}+\frac{3}{4}+...=5,8125$ $S_n=\frac{a(1-r^n)}{1-r}=5,8125$ $\frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}=5,8125$ $6\left[1-\left(\frac{1}{2}\right)^n\right]=5,8125$ $\left(\frac{1}{2}\right)^n=\frac{1}{32}=0,03125$ $2^{-n}=2^{-5}$ or $n\log\frac{1}{2}=\log\frac{1}{32}$ $n=5$ $n=5$	✓ $r=\frac{1}{2}$ ✓substitution ✓simplification ✓answer (4)

3.4	$\sum_{k=1}^{20} 3(2)^{1-k} = p$ $3+\frac{3}{2}+\frac{3}{4}+........+3.2^{-19} = p$ $\sum_{k=1}^{20} 24(2)^{-k}$ $=12+6+3+.....+24.2^{-20}$ $=4\left(3+\frac{3}{2}+\frac{3}{4}+.....+3.2^{-19}\right)$ $=4p$	✓expansion ✓ expansion ✓ answer (3)
	OR/OF	**OR/OF**
	$\sum_{k=1}^{20} 3(2)^{1-k} = p$ $\sum_{k=1}^{20} 6(2)^{-k} = p$ $\therefore \sum_{k=1}^{20} 24(2)^{-k} = 4p$	✓ $\sum_{k=1}^{20} 6(2)^{-k} = p$ ✓ $\sum_{k=1}^{20} 4\times 6(2)^{-k}$ ✓$4p$ (3)
	OR/OF	**OR/OF**
	$\sum_{k=1}^{20} 24(2)^{-k} = \sum_{k=1}^{20} 4\times 3\times 2(2)^{-k}$ $= 4\sum_{k=1}^{20} 3\times 2(2)^{-k}$ $= 4\sum_{k=1}^{20} 3\times (2)^{1-k} = 4p$	✓ $\sum_{k=1}^{20} 4\times 3\times 2(2)^{-k}$ ✓ $4\sum_{k=1}^{20} 3\times 2(2)^{-k}$ ✓$4p$ (3)
	OR/OF	**OR/OF**
	$S_{20} = \frac{3\left(\left(\frac{1}{2}\right)^{20}-1\right)}{\frac{1}{2}-1} = 6 = p$ $S_{20} = \frac{12\left(\left(\frac{1}{2}\right)^{20}-1\right)}{\frac{1}{2}-1} = 24$ $24 = 4\times 6 = 4p$	✓ substitution and answer ✓substitution and answer ✓$4p$ (3)
		[11]

QUESTION/*VRAAG* 4

4.1	Yes For every x-value there is only one corresponding y value **OR/OF** One to one mapping (vertical line test)	✓answer ✓reason (2)
4.2	R(–12 ; –6)	✓answer (1)
4.3	$f(x) = ax^2$ substitute $(-6; -12)$ $-12 = a(-6)^2$ $a = \frac{-1}{3}$	✓substitution ✓answer (2)
4.4	$f : y = -\left(\frac{1}{3}\right)x^2$ $f^{-1} : x = -\left(\frac{1}{3}\right)y^2$ $y^2 = -3x$ $y = \pm\sqrt{-3x}$ Only $y = -\sqrt{-3x}$ and $x \leq 0$	✓swapping x and y ✓ $y^2 = -3x$ ✓ $y = -\sqrt{-3x}$ (3)
		[8]

QUESTION/*VRAAG* 5

5.1	Domain: $x \in R$; $x \neq 1$ **OR/OF** $x \in (-\infty; 1) \cup (1; \infty)$	✓answer (1)
5.2	$x = 1$ $y = 0$	✓$x = 1$ ✓$y = 0$ (2)
5.3		✓ y intercept ✓vertical asymptote ✓shape (3)
5.4	$x \geq 0$; $x \neq 1$ **OR/OF** $0 \leq x < 1$ or $x > 1$ **OR/OF** $x \in [0; 1) \cup (1; \infty)$	✓$x \geq 0$ ✓$x \neq 1$ (2) **OR/OF** ✓ $0 \leq x < 1$ ✓ $x > 1$
		[8]

QUESTION/*VRAAG* 6

6.1	$y = mx + c$ $m = \frac{5-1}{4-0}$ $m = 1$ $c = 1$ $g(x) = x + 1$ **OR/OF** $y = mx + c$ $5 = m(4) + 1$ $m = 1$ $g(x) = x + 1$	✓substitution into gradient formula ✓y-intercept (0 ; 1) (2) **OR/OF** ✓substitute (4 ; 5) ✓$c = 1$ (2)
6.2	$x^2 - 2x - 3 = 0$ $(x+1)(x-3) = 0$ $x = -1$ or $x = 3$ A(–1 ; 0) B(3 ; 0)	✓$y = 0$ ✓factors ✓x-values (3)
6.3	$x = \frac{-1+3}{2}$ or $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)}$ or $f^{/}(x) = 2x - 2 = 0$ $x = 1$ $f(x) = x^2 - 2x - 3$ $y = (1)^2 - 2(1) - 3$ or $y = (x^2 - 2x + (-1)^2) - 3 - 1$ $y = -4$ $= (x-1)^2 - 4$ $y \geq -4$ or $[-4;\infty)$	✓x -value ✓ substitution/ completing the square ✓ answer (3)
6.4.1	**MN:** $y = (x^2 - 2x - 3) - (x+1)$ $= x^2 - 3x - 4$ $6 = x^2 - 3x - 4$ $0 = x^2 - 3x - 10$ $0 = (x-5)(x+2)$ $x = 5$ or $x = -2$ OT = 2 or OT = 5 NA	✓ $x^2 - 3x - 4$ ✓substituting $y = 6$ ✓values of x ✓ OT = 2 (4)
6.4.2	$y = x + 1$ substitute $x = -2$ $= (-2) + 1$ $= -1$ N(–2 ; –1)	✓substituting $x = -2$ ✓answer (2)

6.5	$f'(x)=2x-2$ $2x-2=1$ $x=\frac{3}{2}$ $f\left(\frac{3}{2}\right)=\frac{-15}{4}$ $y+\frac{15}{4}=1\left(x-\frac{3}{2}\right)$ or $-\frac{15}{4}=\frac{1}{2}+c$ $y=x-\frac{21}{4}$ **OR/OF** $x^2-2x-3=x+p$ $x^2-2x-3-x-p=0$ This equation will have equal roots, therefore: $b^2-4ac=0$ $(-3)^2-4(1)(-3-p)=0$ $9+12+4p=0$ $p=\frac{-21}{4}$ $y=x-\frac{21}{4}$	✓ $f'(x)=2x-2$ ✓ $2x-2=1$ ✓ $x=\frac{3}{2}$ ✓ $f\left(\frac{3}{2}\right)=\frac{-15}{4}$ ✓answer (5) **OR/OF** ✓equating ✓equal roots ✓ substitution ✓simplification ✓answer (5)
6.6	$k<\frac{-21}{4}$	✓answer (1)
		[20]

QUESTION/*VRAAG* 7

7.1.1	$F=\frac{x[(1+i)^n-1]}{i}$ $F=\frac{15\ 000\left[\left(1+\frac{0,088}{4}\right)^{16}-1\right]}{\frac{0,088}{4}}$ F = R283 972,28	✓ $\frac{0,088}{4}$ and $n=16$ ✓substitution into correct formula ✓answer (3)
7.1.2	$A = R283\ 972,28 - 100\,000\left(1+\frac{0,088}{4}\right)^4$ = R 174 877,60 **OR/OF** Amount at end of 3 years: $F=\frac{15\ 000\left[\left(1+\frac{0,088}{4}\right)^{12}-1\right]}{\frac{0,088}{4}}-100\,000$ = R103 459,12 Amount at end of 4 years: $P(1+i)^n + \frac{x[(1+i)^n-1]}{i}$ $=103\,459,12\left(1+\frac{0,088}{4}\right)^4+\frac{15000\left[\left(1+\frac{0,088}{4}\right)^4-1\right]}{\frac{0,088}{4}}$ = R 174 877,60	✓ future value – amount including interest ✓$100\,000\left(1+\frac{0,088}{4}\right)^4$ ✓answer (3) **OR/OF** ✓ R15 000 including interest – R100 000 ✓$\left(1+\frac{0,088}{4}\right)^4$ on P and x in F_v ✓method (3)
7.2.1	$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$ $1\ 500\ 000=\frac{x\left[1-\left(1+\frac{0,105}{12}\right)^{-12\times 20}\right]}{\frac{0,105}{12}}$ x = R14 975,70	✓ $i=\frac{0,105}{12}$ ✓ $n=240$ ✓ substitution into correct formula ✓ answer (4)

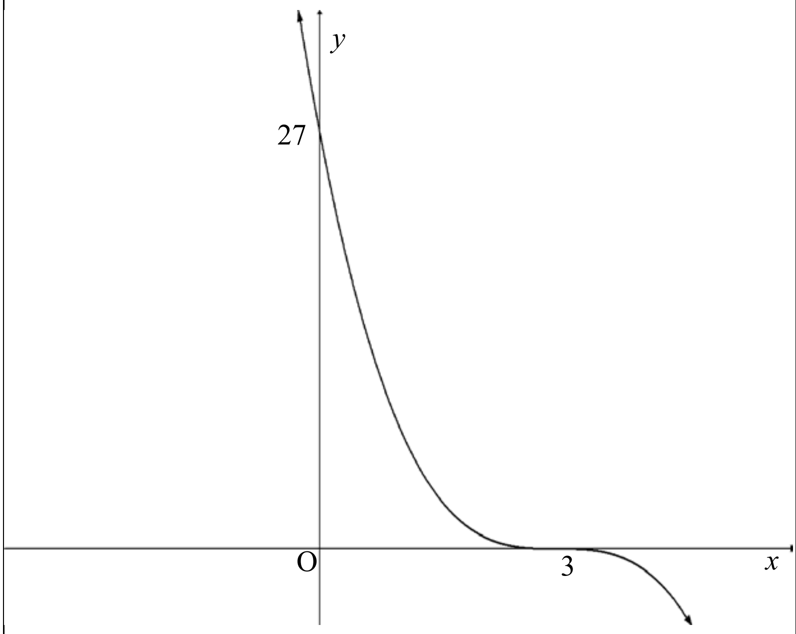
7.2.2	$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P=\frac{14\ 975{,}70\left[1-\left(1+\frac{0{,}105}{12}\right)^{-12\times 8}\right]}{\frac{0{,}105}{12}}$ $P=\text{R}969\,927{,}74$ **OR/OF** Balance outstanding = A – F $=1\ 500\ 000\left(1+\frac{0{,}105}{12}\right)^{144}-\frac{14\ 975{,}70\left[\left(1+\frac{0{,}105}{12}\right)^{144}-1\right]}{\frac{0{,}105}{12}}$ = R5 259 229,61– R4 289 302,47 = R969 927,14	✓R14 975,70 in P_v-formula ✓✓ $n=96$ ✓ substitution into correct formula ✓ answer (5) **OR/OF** ✓ n = 144 in A-formula ✓ $n=144$ in F_v-formula ✓ R14 975,70 ✓ A – F ✓ answer (5)
		[15]

QUESTION/*VRAAG* 8

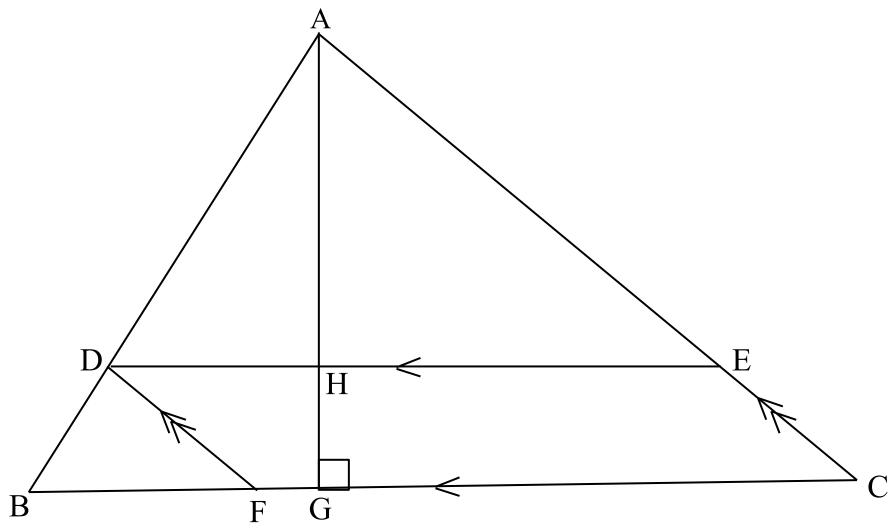
8.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2-5-x^2+5}{h}$ $=\lim_{h\to 0}\frac{h(2x+h)}{h}$ $=\lim_{h\to 0}(2x+h)$ $=2x$	✓ $x^2+2xh+h^2-5$ ✓simplification ✓factorisation ✓ $\lim_{h\to 0}(2x+h)$ ✓ $2x$ (5)
	OR/OF $f(x+h)=(x+h)^2-5$ $=x^2+2xh+h^2-5$ $f(x+h)-f(x)=x^2+2xh+h^2-5-(x^2-5)$ $=2xh+h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2}{h}$ $=\lim_{h\to 0}\frac{h(2x+h)}{h}$ $=\lim_{h\to 0}(2x+h)$ $=2x$	**OR/OF** ✓ $x^2+2xh+h^2-5$ ✓simplification ✓factorisation ✓ $\lim_{h\to 0}(2x+h)$ ✓ $2x$ (5)
8.2.1	$y=3x^3+6x^2+x-4$ $\frac{dy}{dx}=9x^2+12x+1$	✓ $9x^2$ ✓ $12x$ ✓ 1 (3)
8.2.2	$y(x-1)=2x(x-1)$ $y=\frac{2x(x-1)}{x-1}$ if $x\neq 1$ $y=2x$ $\frac{dy}{dx}=2$	✓ $y(x-1)$ ✓ $2x(x-1)$ ✓ $y=2x$ ✓answer (4)
		[12]

QUESTION/*VRAAG* 9

9.1.1	$g(x)=(x+5)(x-x_1)^2$ $20=5(x_1)^2$ ${x_1}^2=4$ $x_1=2$ $g(x)=(x+5)(x-2)^2$ $g(x)=(x+5)(x^2-4x+4)$ $g(x)=x^3+x^2-16x+20$	✓ $(x+5)$ ✓repeated root ✓$x_1=2$ ✓ $g(x)=(x+5)(x^2-4x+4)$ (4)
9.1.2	$g(x)=x^3+x^2-16x+20$ $g'(x)=3x^2+2x-16$ $3x^2+2x-16=0$ $(3x+8)(x-2)=0$ $x=\frac{-8}{3}$ or $x=2$ $\text{R}\left(\frac{-8}{3};\frac{1372}{27}\right)$ or $\text{R}(-2{,}67;50{,}81)$ $\text{P}(2;0)$	✓derivative ✓equating to zero ✓factors ✓co-ordinates of R ✓ co-ordinates of P (5)
9.1.3	$g''(x)=6x+2$ $g''(0)=2$ $\therefore$ concave up **OR/OF** $g''(x)=6x+2$ $6x+2=0$ $x=-\frac{1}{3}$ is the point of inflection $\therefore$ concave up	✓ $g''(x)=6x+2$ ✓ $g''(0)=2$ ✓conclusion (3) **OR/OF** ✓ $g''(x)=6x+2$ ✓ $x=-\frac{1}{3}$ ✓conclusion (3)

9.2		✓ y – intercept of a cubic graph ✓ point of inflection and stationary point, $x = 3$ ✓ concave up for $x < 3$ and concave down for $x > 3$ (3)
		[15]

QUESTION/*VRAAG* 10

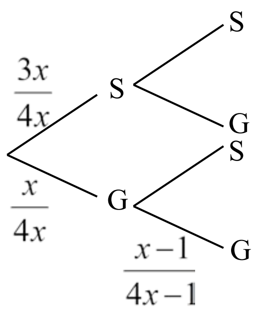


10.1	$\frac{AH}{HG}=\frac{3}{2}$	✓ answer (1)
10.2	Area of a parallelogram $=$ base $\times \perp$ height Area $=\frac{3}{5}(5-t).\frac{2}{5}t$ Area $=\frac{6}{25}(5-t)t$ $A(t)=-\frac{6}{25}t^2+\frac{6}{5}t$ $A'(t)=-\frac{12}{25}t+\frac{6}{5}$ $-\frac{12}{25}t+\frac{6}{5}=0$ $12t-30=0$ $t=\frac{30}{12}$ or $\frac{5}{2}$	✓ $\frac{2}{5}t$ ✓ $\frac{3}{5}(5-t)$ ✓ $A(t)=-\frac{6}{25}t^2+\frac{6}{5}t$ ✓ $-\frac{12}{25}t+\frac{6}{5}$ ✓answer (5)
		[6]

QUESTION/*VRAAG* 11

11.1.1	$7^5 = 16\ 807$	✓✓ answer (2)
11.1.2	$7 \times 6 \times 5 \times 4 \times 3$ $= \frac{7!}{2!} = 2520$	✓$7 \times 6 \times 5 \times 4 \times 3$ or $\frac{7!}{2!}$ ✓answer (2)
11.2	$2 \times 7 \times 1 = 14$	✓✓✓ $2 \times 7 \times 1$ (3)
		[7]

QUESTION/*VRAAG* 12

12.1	P(A or B) = P(A) + P(B) $0{,}74 = 0{,}45 + y$ $y = 0{,}29$	✓ P(A or B) = P(A) + P(B) ✓substitution ✓answer (3)
12.2	 Let the number of mystery gift bags = x The total number of bags = $4x$ $\left(\frac{x}{4x}\right) \times \left(\frac{x-1}{4x-1}\right) = \frac{7}{118}$ $\frac{1}{4} \times \frac{x-1}{4x-1} = \frac{7}{118}$ $\frac{x-1}{4x-1} = \frac{28}{118}$ $118x - 118 = 112x - 28$ $x = 15$	✓ $4x$ ✓$\left(\frac{x}{4x}\right)$ or $\left(\frac{1}{4}\right)$ ✓$\left(\frac{x-1}{4x-1}\right)$ ✓$\frac{1}{4} \times \frac{x-1}{4x-1}$ ✓ equating to $\frac{7}{118}$ ✓answer (6)

	OR/OF P(gift and gift) = P(gift at first draw) × P(gift at second draw) $\frac{7}{118}=\frac{1}{4}\times \text{P(gift at second draw)}$ P(gift at second draw) = $\frac{7}{118}\div\frac{1}{4}$ $=\frac{14}{59}$ Therefore: P(gift at first draw) $=\frac{15}{60}$ And: 15 bags had mystery gifts inside	**OR/OF** ✓ $\frac{1}{4}$ ✓ $\frac{1}{4}\times \text{P(gift at } 2^{nd} \text{ draw)}$ ✓ $\frac{7}{118}=\frac{1}{4}\times \text{P(gift at } 2^{nd} \text{ draw)}$ ✓ $\frac{14}{59}$ ✓ $\frac{15}{60}$ ✓answer (6)
		[9]

TOTAL/*TOTAAL*: 150

MATHEMATICS P1

2019

QUESTION PAPER

Inside Education
Inspiring Minds

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 5x - 6 = 0$ (3)

1.1.2 $4x^2 + 3x - 5 = 0$ (correct to TWO decimal places) (3)

1.1.3 $4x^2 - 1 < 0$ (3)

1.1.4 $\left(\sqrt{\sqrt{32}+x}\right)\left(\sqrt{\sqrt{32}-x}\right) = x$ (4)

1.2 Solve simultaneously for x and y:
$y + x = 12$ and $xy = 14 - 3x$ (5)

1.3 Consider the product $1 \times 2 \times 3 \times 4 \times \ldots \times 30$.
Determine the largest value of k such that 3^k is a factor of this product. (4)

[22]

QUESTION 2

2.1 Given the quadratic sequence: 321 ; 290 ; 261 ; 234 ; ....

2.1.1 Write down the values of the next TWO terms of the sequence. (2)

2.1.2 Determine the general term of the sequence in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Which term(s) of the sequence will have a value of 74? (4)

2.1.4 Which term in the sequence has the least value? (2)

2.2 Given the geometric series: $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + \ldots = \text{K}$

2.2.1 Determine the value of K if the series has 21 terms. (3)

2.2.2 Determine the largest value of n for which $T_n > \frac{5}{8192}$ (4)

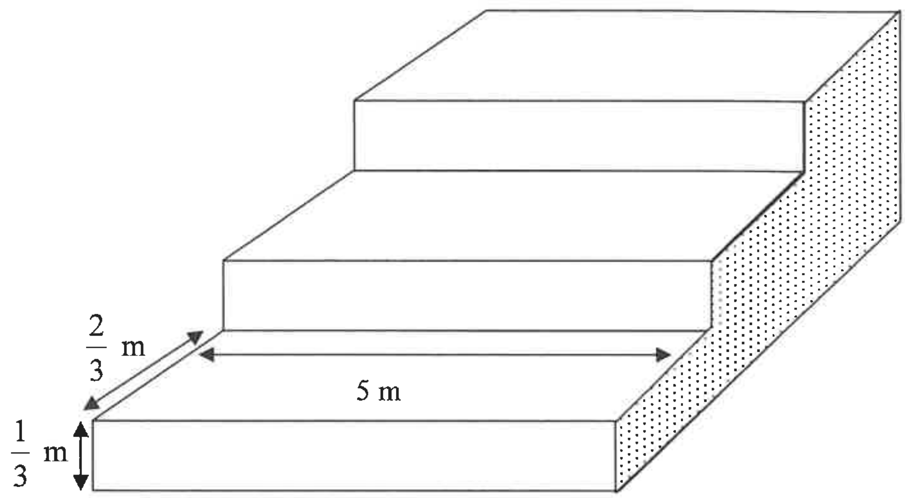
[19]

QUESTION 3

3.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$ (3)

3.2 A steel pavilion at a sports ground comprises of a series of 12 steps, of which the first 3 are shown in the diagram below.

Each step is 5 m wide. Each step has a rise of $\frac{1}{3}$ m and has a tread of $\frac{2}{3}$ m, as shown in the diagram below.



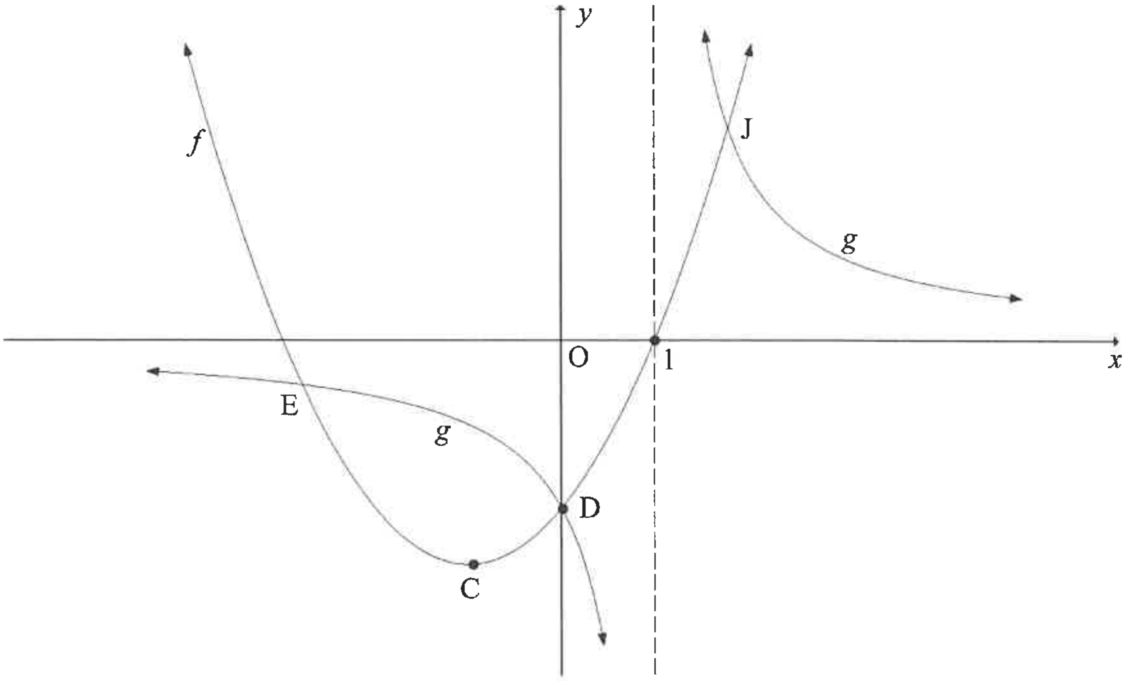
The open side (shaded on sketch) on each side of the pavilion must be covered with metal sheeting. Calculate the area (in m^2) of metal sheeting needed to cover both open sides. (6)

[9]

QUESTION 4

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \dfrac{a}{x+p}$.

- f has a turning point at C and passes through the x-axis at $(1\ ;0)$.
- D is the y-intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



4.1 Write down the value of p. (1)

4.2 Show that $a = 3$ and $b = 2$. (3)

4.3 Calculate the coordinates of C. (4)

4.4 Write down the range of f. (2)

4.5 Determine the equation of the line through C that makes an angle of $45°$ with the positive x-axis. Write your answer in the form $y = \ldots$ (3)

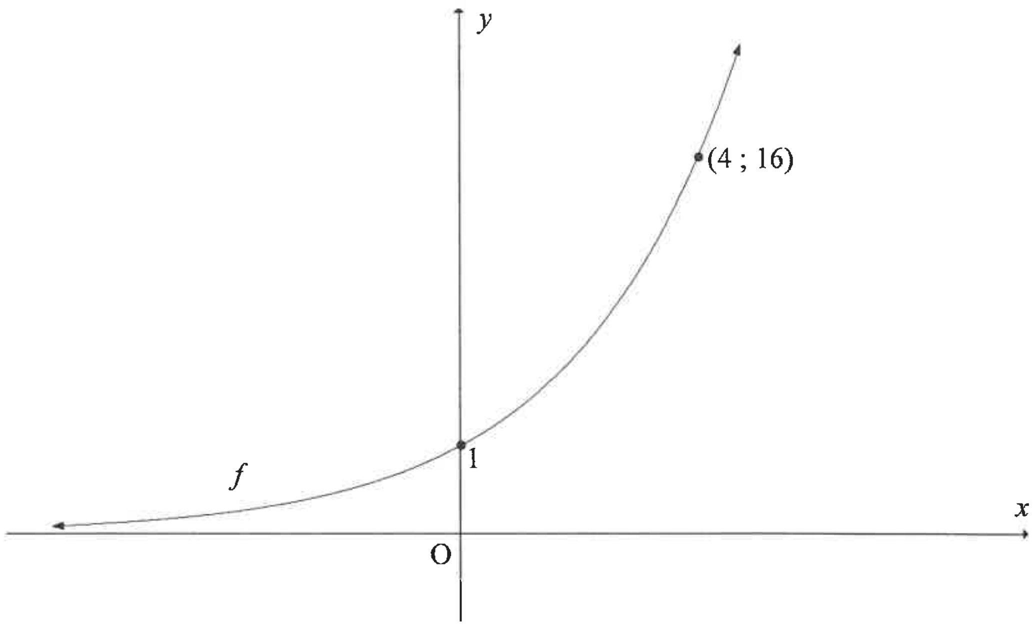
4.6 Is the straight line, determined in QUESTION 4.5, a tangent to f? Explain your answer. (2)

4.7 The function $h(x) = f(m - x) + q$ has only one x-intercept at $x = 0$. Determine the values of m and q. (4)

[19]

QUESTION 5

Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point (4 ; 16) lies on f.



5.1 Determine the value of k. (2)

5.2 Graph g is obtained by reflecting graph f about the line $y = x$. Determine the equation of g in the form $y = \ldots$ (2)

5.3 Sketch the graph g. Indicate on your graph the coordinates of two points on g. (4)

5.4 Use your graph to determine the value(s) of x for which:

5.4.1 $f(x) \times g(x) > 0$ (2)

5.4.2 $g(x) \leq -1$ (2)

5.5 If $h(x) = f(-x)$, calculate the value of x for which $\text{f(x)} - \text{h(x)} = \frac{15}{4}$ (4)

[16]

QUESTION 6

6.1 Two friends, Kuda and Thabo, each want to invest R5 000 for four years. Kuda invests his money in an account that pays simple interest at 8,3% per annum. At the end of four years, he will receive a bonus of exactly 4% of the accumulated amount. Thabo invests his money in an account that pays interest at 8,1% p.a., compounded monthly.

Whose investment will yield a better return at the end of four years? Justify your answer with appropriate calculations. (5)

6.2 Nine years ago, a bank granted Mandy a home loan of R525 000. This loan was to be repaid over 20 years at an interest rate of 10% p.a., compounded monthly. Mandy's monthly repayments commenced exactly one month after the loan was granted.

6.2.1 Mandy decided to make monthly repayments of R6 000 instead of the required R5 066,36. How many payments will she make to settle the loan? (5)

6.2.2 After making monthly repayments of R6 000 for nine years, Mandy required money to fund her daughter's university fees. She approached the bank for another loan. Instead, the bank advised Mandy that the extra amount repaid every month could be regarded as an investment and that she could withdraw this full amount to fund her daughter's studies. Calculate the maximum amount that Mandy may withdraw from the loan account. (4)

[14]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 4 - 7x$. (4)

7.2 Determine $\frac{dy}{dx}$ if $y = 4x^8 + \sqrt{x^3}$ (3)

7.3 Given: $y = ax^2 + a$

Determine:

7.3.1 $\frac{dy}{dx}$ (1)

7.3.2 $\frac{dy}{da}$ (2)

7.4 The curve with equation $y = x + \frac{12}{x}$ passes through the point A(2 ; b). Determine the equation of the line perpendicular to the tangent to the curve at A. (4)

[14]

QUESTION 8

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t)=(t-6)(-2t^2+3t-6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

8.1 At what height above the floor did the insect start to crawl? (1)

8.2 How many times did the insect reach the floor? (3)

8.3 Determine the maximum height that the insect reached above the floor. (4)
[8]

QUESTION 9

Given: $f(x)=3x^3$

9.1 Solve $f(x)=f'(x)$ (3)

9.2 The graphs f, f' and f'' all pass through the point $(0\,;0)$.

9.2.1 For which of the graphs will $(0\,;0)$ be a stationary point? (1)

9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)

9.3 Determine the vertical distance between the graphs of f' and f'' at $x=1$. (3)

9.4 For which value(s) of x is $f(x)-f'(x)<0$? (4)
[13]

QUESTION 10

The school library is open from Monday to Thursday. Anna and Ben both studied in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on:

10.1 The same day (2)

10.2 Consecutive days (3)
[5]

QUESTION 11

11.1 Events **A** and **B** are independent. P(A) = 0,4 and P(B) = 0,25.

11.1.1 Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region. (3)

11.1.2 Determine P(**A** or NOT **B**). (2)

11.2 Motors Incorporated manufacture cars with 5 different body styles, 4 different interior colours and 6 different exterior colours, as indicated in the table below.

BODY STYLES	INTERIOR COLOURS	EXTERIOR COLOURS
Five body styles	Blue	Silver
	Grey	Blue
		White
	Black	Green
	Red	Red
		Gold

The interior colour of the car must NOT be the same as the exterior colour.

Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has a floor space of $500\ m^2$ and each car requires a floor space of $5\,m^2$.

Is this display possible? Justify your answer with the necessary calculations. (6)

[11]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}\ ;\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad \mathrm{P(A\ or\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ and\ B)}$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$